Question 1

Marks

(a) Solve for t: $7-4t>12$. **2**

(b) Simplify: $\dfrac{x}{2}-\dfrac{2x+5}{6}$. **2**

(c) Solve $2\sin\theta=-1$ for $0\le\theta\le 2\pi$. **2**

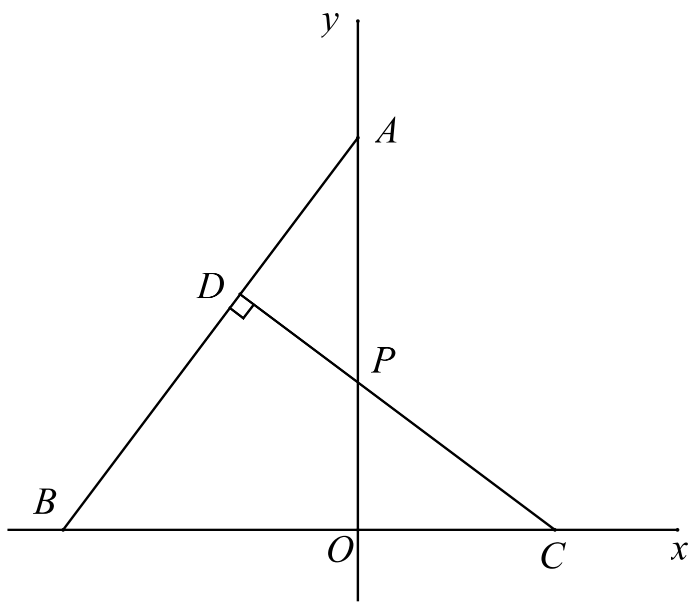
(d) Differentiate with respect to x:

(i) $y=\dfrac{6}{\sqrt{x}}$. **1**

(ii) $y=x^2\ln x$. **2**

(e) Evaluate $\displaystyle\int_1^5\left(3+\frac{2}{x}\right)dx$. **3**

Question 2 (START A NEW PAGE)



In the diagram $AB=BC$ and CD is perpendicular to AB. CD intersects the y-axis at P.

(i) Find the length of AB. **1**

(ii) Hence show that the co-ordinates of point C are $(2,0)$. **1**

(iii) Show that the equation of CD is $3x+4y=6$. **3**

(iv) Show that the co-ordinates of P are $\left(0,1\tfrac{1}{2}\right)$. **1**

(v) Show that the length of CP is $2\tfrac{1}{2}$. **1**

(vi) Prove that ΔADP is congruent to ΔCOP **3**

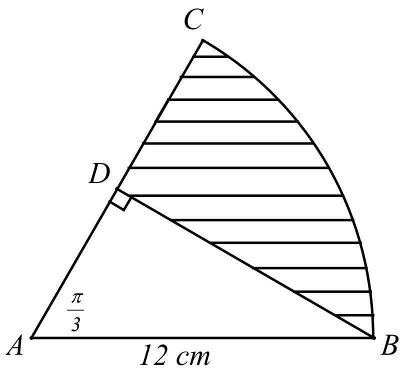
(vii) Hence calculate the area of the quadrilateral $DPOB$. **2**

Question 3 (START A NEW PAGE)

Marks

(a) Find the equation of the tangent to the curve $y = x^3 - 4x - 1$ at the point $T(2,-1)$. **3**

(b)



ABC is a sector with $\angle BAC = \dfrac{\pi}{3}$ and $AC = AB = 12\ cm$.

(i) Calculate the area of sector ABC. **1**

(ii) Calculate the area of the shaded region. **3**

(c) (i) The equation of a parabola is given as $y = \frac{1}{8}x^2 - x$. Rewrite the equation in the form $(x - x_o)^2 = b(y + y_o)$ where x_o, y_o and b are constants. **2**

(ii) Hence write down the focal length of the parabola and the co-ordinates of its vertex and focus. **3**

Question 4 (START A NEW PAGE)

(a) The price of gold, \$$P$, was studied over a period of t years.

(i) Throughout the period of study $\dfrac{dP}{dt} > 0$. What does this say about the price of gold? **1**

(ii) It was also observed that the rate of change of the price of gold decreased over the period of study. **1**

What does this statement imply about $\dfrac{d^2P}{dt^2}$?

(b) A pool is being drained at a rate of $240t - 9600$ litres/minute.

(i) How long does it take before the draining of the pool stops? **1**

(ii) If the pool initially held 1920000 litres of water, find the volume of water in the pool after 15 minutes. **3**

(c) In a hat are 5 red and 3 green jellybeans. Jason reaches into the hat and randomly selects two jellybeans. Find the probability that:

(i) both selected jellybeans are red. **1**

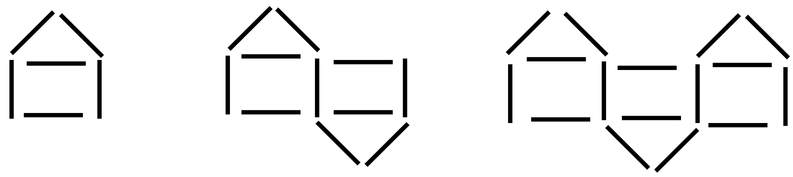
(ii) the chosen jellybeans are different colours. **2**

(d) The tangent to the curve $y = 4\sqrt{x}$ at a point P has a slope of 2. Find the co-ordinates of point P. **3**

Question 5 (START A NEW PAGE)

Marks

(a)



Pattern #1 *Pattern #2* *Pattern #3*

The above patterns are made using small sticks. Pattern #1 requires 6 sticks, pattern #2 requires 11 sticks and pattern #3 requires 16 sticks.

(i) Write a formula for the number of sticks, U_n, needed to construct pattern number n. **2**

(ii) What is the largest pattern that can be constructed from 200 sticks? **1**

(iii) How many sticks would be needed to construct all the patterns from pattern #1 to pattern #20? **2**

(b) (i) Show that the curves $y = 1 + \sqrt{x}$ and $y = 7 - x$ meet at the point $(4, 3)$. **1**

(ii) Sketch, on the same set of axes, the curves $y = 1 + \sqrt{x}$ and $y = 7 - x$ and shade the region bounded by these curves and the y-axis. **2**

(iii) Find the volume of the solid formed when the area bounded by the $y = 1 + \sqrt{x}$, $y = 7 - x$ and the y-axis is rotated one revolution about the y-axis. **4**

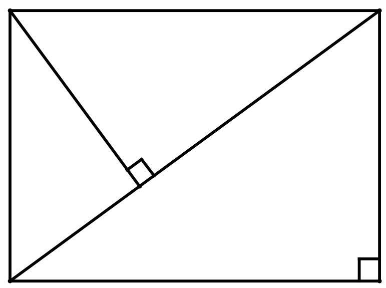
Question 6 (START A NEW PAGE)

(a) (i) Show that the curves $y = \sin 2x$ and $y = \cos x$ meet at the point with abscissa $x = \frac{\pi}{6}$. **1**

(ii) Sketch, on the same set of axes, the curves $y = \sin 2x$ and $y = \cos x$ in the domain $0 \le x \le \frac{\pi}{2}$ and shade the region bounded only by these curves. **2**

(iii) Find the area of the shaded region described in (ii). **3**

(b)



(Diagram not drawn to scale)

$ABCD$ is a rectangle with $AB = 12\ cm$, $BC = 9\ cm$ and AM perpendicular to BD.

(i) Copy the diagram onto your answer sheet and find the length of BD. **1**

(ii) Prove that ΔABM is similar to ΔBDC. **3**

(iii) Hence find the length of BM. (Give reasons) **2**

Question 7 (START A NEW PAGE)

Marks

(a) In an experiment, the amount of crystals, x grams, that dissolved in a solution after t minutes was given by:

$$x = 200\left(1 - e^{-kt}\right)$$

(i) After 3 minutes it was found that 50 grams of crystals had dissolved, hence find the value of k. **2**

(ii) At what rate were the crystals dissolving after 5 minutes? Give your answer to the nearest gram/minute. **3**

(b) Two particles A and B start from the origin at the same time and move along a straight line so that their velocities in m/s at any time t seconds are given by:

$$v_A = t^2 + 2 \text{ and } v_B = 8 - 2t$$

(i) Show that the two particles never move with the same acceleration. **3**

(ii) Write a formula for the position of particle A at time t seconds. **2**

(iii) After leaving the origin, find when the two particles are again at the same position. **2**

Question 8 (START A NEW PAGE)

(i) Given that $f(x) = x^2\sqrt{10-x}$, show that $f'(x) = \dfrac{5x(8-x)}{2\sqrt{10-x}}$. **3**

(ii) State the domain of the function $y = x^2\sqrt{10-x}$. **1**

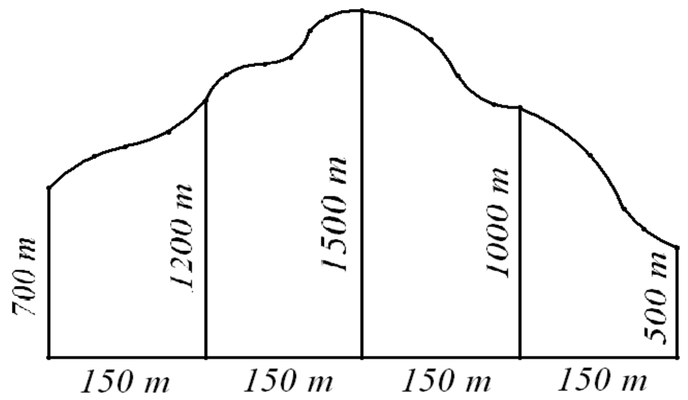
(iii) Find all the stationary points on the curve $y = x^2\sqrt{10-x}$ and determine their nature. **5**

(iv) Sketch the curve $y = x^2\sqrt{10-x}$ indicating its intercepts with the co-ordinate axes and the position of all stationary points. **3**

Question 9 (START A NEW PAGE)

Marks

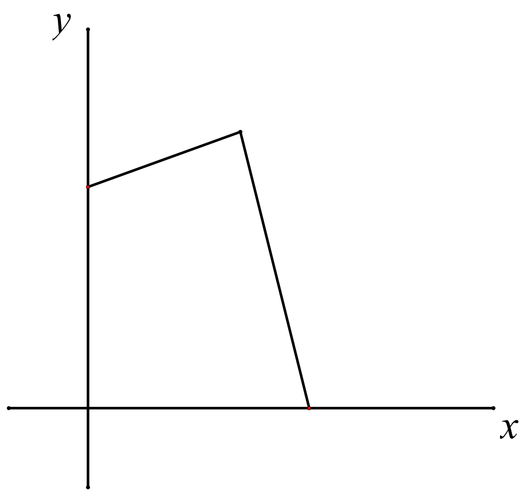
(a) Prime land along a foreshore is to be reclaimed and developed as part of a housing estate. A plan of the land to be reclaimed is shown below: **3**



(Diagram not drawn to scale)

Use Simpson's rule with five function values to find the approximate area, expressed in hectares, of the land to be reclaimed.

(b)



The point $P(x,y)$ moves so that its distance from the point $M(3,0)$ is always twice its distance from the point $N(0,3)$.

(i) Show that the equation of the locus of the point $P(x,y)$ is $x^2+2x+y^2-8y+9=0$. **2**

(ii) Give a geometric description of the locus. **2**

(c) (i) Graph $y=f(x)$ if $f(x)=\begin{cases}-4x & \text{for } x<0\\ x^2+3 & \text{for } x\geq 0\end{cases}$ **2**

(ii) Evaluate $f(-2)+f(2)$. **1**

(iii) Solve for $a>0: f(a)=f(-a)$. **2**

Question 10 (START A NEW PAGE)

Marks

(a) Abigale decides to save for a car. She creates a savings account into which she deposits \$100 at the beginning of each week. The account pays interest at a rate of 0.16% per week calculated on the value of the account at the end of each week.

(i) Show that Abigale's account is worth $62600\left(1.0016^n - 1\right)$ dollars at the end of n weeks. **3**

(ii) Find the value of Abigale's account at the end of 52 weeks. Give your answer correct to the nearest dollar. **1**

(iii) Find the number of weeks needed for Abigale's account to accumulate \$20000. (Give your answer to nearest week) **3**

(b) When a ship is travelling at a speed of v km/hr, its rate of consumption of fuel in tonnes per hour is given by $125 + 0.004v^3$.

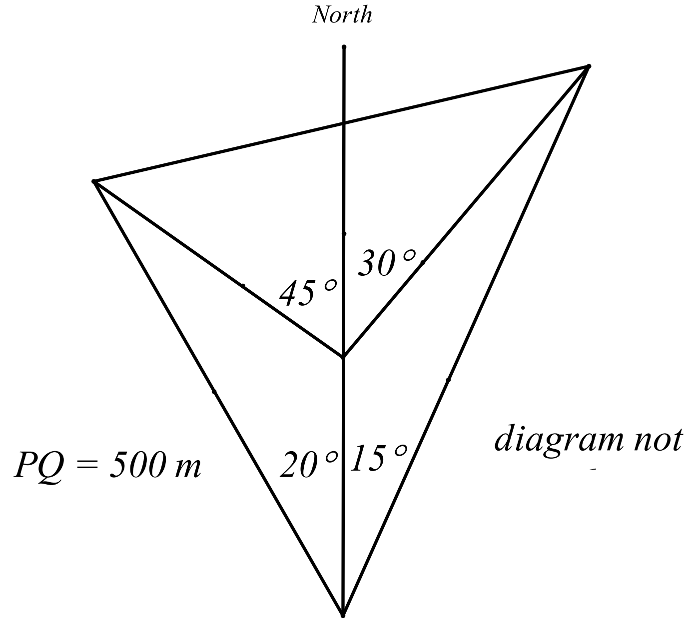
(i) Show that on a voyage of 5000km at a speed of v km/hr the formula for the total fuel used, T tonnes, is given by: **1**

$$T = \frac{625000}{v} + 20v^2$$

(ii) Hence find the speed for the greatest fuel economy and the amount of fuel used at this speed. (Justify your answer.) **4**

☯ This is the END of the examination paper ☝

(b)



A tourist at point P sees two towers at X and Y on bearings of $340°T$ and $015°T$ respectively. The tourist then walks 500 metres due north point Q and now records the bearings as $315°T$ and $030°T$ respectively.

(i)	Show that $XP = \dfrac{250\sqrt{2}}{\sin 25°}$.	2
(ii)	Show that $YP = \dfrac{250\sqrt{3}}{\sin 15°}$.	2
(iii)	Find the distance from tower X to tower Y correct to the nearest 10 metres.	2

Curve Sketching

Consider the curve given by the equation:

$$y = 0.4x(x-6)^2$$

(i)	Find the co-ordinates of the stationary points and determine their nature.	4
(ii)	Sketch the curve $y = 0.4x(x-6)^2$ in the domain $-2 \le x \le 10$, clearly showing the intercepts with the co-ordinate axes, the stationary points and end points.	3
(iii)	What is the maximum value of $0.4x(x-6)^2$ in the domain $-2 \le x \le 10$?	1

Year 12 Trial HSC Examination - Mathematics (2U) 2008 - SOLUTIONS

Question 1

Marks

(a) Solve for t: $7-4t>12$. **2**

$-4t>5$

$t<-1\frac{1}{4}$

(b) Simplify: $\frac{x}{2}-\frac{2x+5}{6}$. **2**

$$\frac{x}{2}-\frac{2x+5}{6}=\frac{3x}{6}-\frac{2x+5}{6}$$

$$=\frac{3x-2x-5}{6}$$

$$=\frac{x-5}{6}$$

(c) Solve $2\sin\theta=-1$ for $0\le\theta\le 2\pi$. **2**

$\sin\theta=-\frac{1}{2}$

$\theta=\pi+\frac{\pi}{6}$ or $2\pi-\frac{\pi}{6}$

$\theta=\frac{7\pi}{6}$ or $\frac{11\pi}{6}$

(d) Differentiate with respect to x:

(i) $y=\frac{6}{\sqrt{x}}$. **1**

$$y=6x^{-\frac{1}{2}}$$

$$y'=-3x^{-1\frac{1}{2}}$$

$$=-\frac{3}{x\sqrt{x}}$$

(ii) $y=x^2\ln x$. **2**

$$y'=2x\ln x+x^2.\frac{1}{x}$$

$$=2x\ln x+x$$

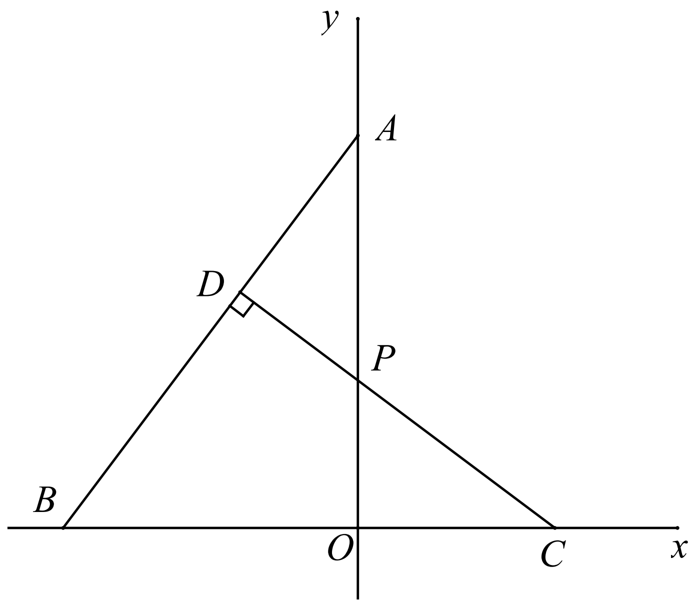
(e) Evaluate $\int_1^5\left(3+\frac{2}{x}\right)dx$. **3**

$$\int_1^5\left(3+\frac{2}{x}\right)dx=\left[3x+2\ln x\right]_1^5$$

$$=(15+2\ln 5)-(3+2\ln 1)$$

$$=12+2\ln 5$$

Question 2 (START A NEW PAGE)



In the diagram $AB = BC$ and CD is perpendicular to AB. CD intersects the y-axis at P.

(i) Find the length of AB. **1**

$AB^2 = 3^2 + 4^2$ (Pythagoras' Theorem)

$AB = 5$

(ii) Hence show that the co-ordinates of point C are $(2, 0)$. **1**

$BC = AB$

$\therefore BC = 5$

$OB + OC = 5$

$3 + OC = 5$

$OC = 2$

$\therefore C$ is $(2,0)$

(iii) Show that the equation of CD is $3x + 4y = 6$. **3**

$m(AB) = \frac{4}{3}$

$m(CD) = -\frac{3}{4}$

Eqn. CD : $y - 0 = -\frac{3}{4}(x - 2)$

$4y = -3x + 6$

$3x + 4y = 6$

(iv) Show that the co-ordinates of P are $(0, 1\frac{1}{2})$. **1**

$3x + 4y = 6$

at P $x = 0$

$\therefore 0 + 4y = 6$

$y = 1\frac{1}{2}$

(v) Show that the length of CP is $2\frac{1}{2}$. **1**

$$CP^2 = \left(1\tfrac{1}{2}\right)^2 + 2^2 \quad \text{(Pythagoras' Theorem)}$$
$$= 6\tfrac{1}{4}$$
$$CP = 2\tfrac{1}{2}$$

(vi) Prove that ΔADP is congruent to ΔCOP **3**

In ΔADP and ΔCOP

$\angle ADP = \angle POC$ (both 90°)

$\angle DPA = \angle OPC$ (vertically opposite angles are equal)

$AP = PC$ (both $= 2\frac{1}{2}$)

$\therefore \Delta ADP \equiv \Delta COP (AAS)$

(vii) Hence calculate the area of the quadrilateral $DPOB$. **2**

$$\text{area} BDPO = \text{area } \Delta BAO - \text{area } \Delta ADP$$
$$= \text{area } \Delta BAO - \text{area } \Delta OPC \quad (\Delta OPC \equiv \Delta ADP)$$
$$= \tfrac{1}{2}(3)(4) - \tfrac{1}{2}(2)\left(1\tfrac{1}{2}\right) \ u^2$$
$$= 4\tfrac{1}{2} \ u^2$$

Question 3 (START A NEW PAGE)

Marks

(a) Find the equation of the tangent to the curve $y = x^3 - 4x - 1$ at the point $T(2,-1)$. **3**

$y' = 3x^2 - 4$

when $x = 2$, $y' = 3(2)^2 - 4$

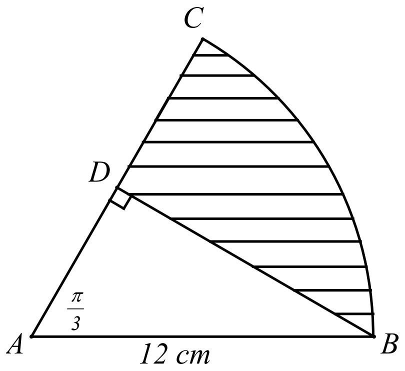
$= 8$

eqn. of tangent

$y + 1 = 8(x - 2)$

$y = 8x - 17$

(b)



ABC is a sector with $\angle BAC = \dfrac{\pi}{3}$ and $AC = AB = 12\ cm$.

(i) Calculate the area of sector ABC. **1**

$\text{area} = \frac{1}{2}(12)^2.\left(\frac{\pi}{3}\right)\ cm^2$

$= 24\pi\ cm^2$

(ii) Calculate the area of the shaded region. **3**

$\dfrac{AD}{12} = \cos\left(\frac{\pi}{3}\right)\ cm$

$AD = 12 \times \dfrac{1}{2}\ cm$

$= 6\ cm$

$\text{area } \Delta ADB = \dfrac{1}{2} \times 6 \times 12 \times \sin\left(\frac{\pi}{3}\right)\ cm^2$

$= 36 \times \dfrac{\sqrt{3}}{2}\ cm^2$

$= 18\sqrt{3}\ cm^2$

$\text{shaded area} = \left(24\pi - 18\sqrt{3}\right)\ cm^2$

(c) (i) The equation of a parabola is given as $y = \frac{1}{8}x^2 - x$. Rewrite the equation in the form $(x - x_o)^2 = b(y + y_o)$ where x_o, y_o and b are constants. **2**

$$y = \tfrac{1}{8}x^2 - x$$
$$8y = x^2 - 8x$$
$$8y + 16 = x^2 - 8x + 16$$
$$(x-4)^2 = 8(y+2)$$

(ii) Hence write down the focal length of the parabola and the co-ordinates of its vertex and focus. **3**

focal length = 2
vertex $(4,-2)$
focus $(4,0)$

Question 4 (START A NEW PAGE)

(a) The price of gold, \$$P$, was studied over a period of t years.

(i) Throughout the period of study $\frac{dP}{dt} > 0$. What does this say about the price of gold? **1**

Price of gold is increasing

(ii) It was also observed that the rate of change of the price of gold decreased over the period of study. **1**

What does this statement imply about $\frac{d^2P}{dt^2}$?

$$\frac{d^2P}{dt^2} < 0$$

(b) A pool is being drained at a rate of $240t - 9600$ litres/minute.

(i) How long does it take before the draining of the pool stops? **1**

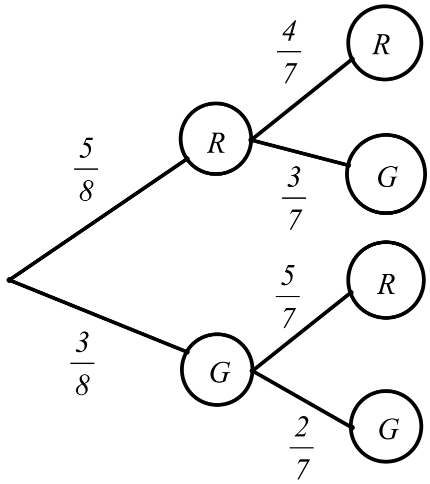
$$240t - 9600 = 0$$
$$t = \frac{9600}{240}$$
$$= 40$$
time = 40minutes

(ii) If the pool initially held 1920000 litres of water, find the volume of water in the pool after 15 minutes. **3**

$$\frac{dV}{dt} = 240t - 9600$$
$$V = 120t^2 - 9600t + c$$
when $t = 0, V = 1920000$
$$1920000 = 0 - 0 + c$$
$$c = 1920000$$
$$V = 120t^2 - 9600t + 1920000$$
when $t = 15$
$$V = 120(15)^2 - 9600(15) + 1920000$$
$$= 1803000$$
volume $= 1803000\ l$

(c) In a hat are 5 red and 3 green jellybeans. Jason reaches into the hat and randomly selects two jellybeans. Find the probability that:

(i) both selected jellybeans are red. **1**



$$\mathrm{P}(\text{Red},\text{Red}) = \frac{5}{8}\times\frac{4}{7} = \frac{5}{14}$$

(ii) the chosen jellybeans are different colours. **2**

$$\begin{aligned}\mathrm{P}(\text{different}) &= \mathrm{P}(\mathrm{R},\mathrm{G}) + \mathrm{P}(\mathrm{G},\mathrm{R})\\ &= \frac{5}{8}\times\frac{3}{7}+\frac{3}{8}\times\frac{5}{7}\\ &= \frac{15}{28}\end{aligned}$$

(d) The tangent to the curve $y = 4\sqrt{x}$ at a point P has a slope of 2. Find the co-ordinates of point P. **3**

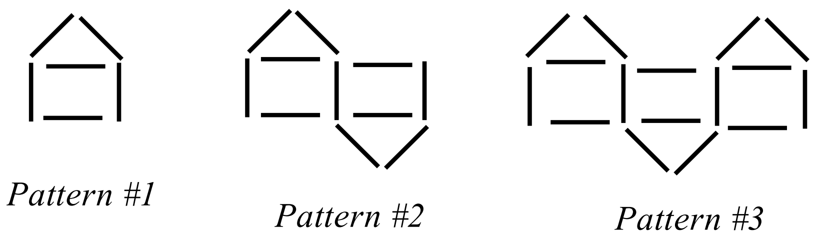
$$\begin{aligned} y &= 4x^{\frac{1}{2}}\\ y' &= 2x^{-\frac{1}{2}}\\ &= \frac{2}{\sqrt{x}}\\ \therefore 2 &= \frac{2}{\sqrt{x}}\\ \sqrt{x} &= 1\\ x &= 1\\ y &= 4\sqrt{1}\\ y &= 4 \end{aligned}$$

P is $(1,4)$

Question 5 **(START A NEW PAGE)**

Marks

(a)



The above patterns are made using small sticks. Pattern #1 requires 6 sticks, pattern #2 requires 11 sticks and pattern #3 requires 16 sticks.

(i) Write a formula for the number of sticks, U_n, needed to construct pattern number n. **2**

$U_n = 6 + 5(n-1)$

$U_n = 5n + 1$

(ii) What is the largest pattern that can be constructed from 200 sticks? **1**

$5n + 1 = 200$

$n = 39.8$

$\therefore$ can build pattern number 39

(iii) How many sticks would be needed to construct all the patterns from pattern #1 to pattern #20? **2**

$S_{20} = \frac{20}{2}\{2(6) + 5(19)\}$

$= 1070$

$\therefore$ need 1070 sticks

(b) (i) Show that the curves $y = 1 + \sqrt{x}$ and $y = 7 - x$ meet at the point $(4, 3)$. **1**

when $x = 4$, $y = 1 + \sqrt{x}$

$= 1 + \sqrt{4}$

$= 3$

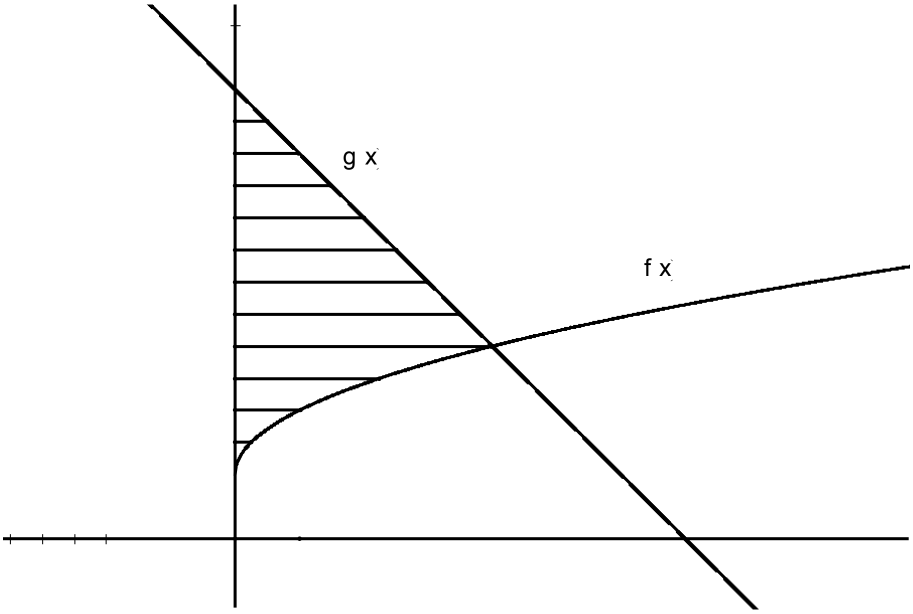
when $x = 4$, $y = 7 - x$

$= 7 - 4$

$= 3$

$\therefore$ point $(4,3)$ lies on both curves

(ii) Sketch, on the same set of axes, the curves $y = 1 + \sqrt{x}$ and $y = 7 - x$ and shade the region bounded by these curves and the y-axis. **2**



(iii) Find the volume of the solid formed when the area bounded by the $y = 1 + \sqrt{x}$, $y = 7 - x$ and the y-axis is rotated one revolution about the y-axis. **4**

Volume = Vol. of revolution of $y = 1 + \sqrt{x}$ for $1 \le y \le 3$ + Vol. of cone for $3 \le y \le 7$

$$\text{Volume} = \pi \int_1^3 x^2 \, dy + \frac{1}{3}\pi(4)^2(4) \; u^3$$

$$= \pi \int_1^3 (y-1)^4 \, dy + \frac{64\pi}{3} \; u^3$$

$$= \pi \left[\frac{(y-1)^5}{5} \right]_1^3 + \frac{64\pi}{3} \; u^3$$

$$= \pi \left\{ \frac{32}{5} - 0 \right\} + \frac{64\pi}{3} \; u^3$$

$$\text{Volume } = \frac{416\pi}{15} \; u^3$$

Question 6 (START A NEW PAGE)

(a) (i) Show that the curves $y=\sin 2x$ and $y=\cos x$ meet at the point with abscissa $x=\frac{\pi}{6}$. **1**

when $x=\frac{\pi}{6}, y=\sin\left(\frac{2\pi}{6}\right)$

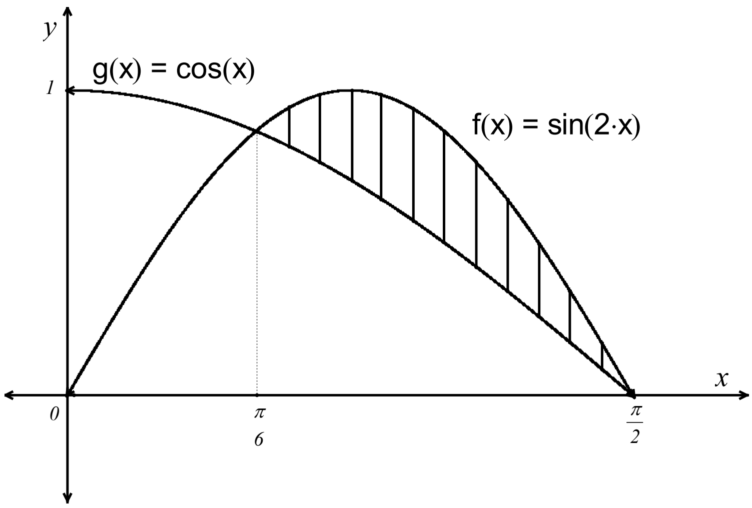
$=\frac{\sqrt{3}}{2}$

when $x=\frac{\pi}{6}, y=\cos\left(\frac{\pi}{6}\right)$

$=\frac{\sqrt{3}}{2}$

$\therefore$ point $\left(\frac{\pi}{6},\frac{\sqrt{3}}{2}\right)$ lies on both curves

(ii) Sketch, on the same set of axes, the curves $y=\sin 2x$ and $y=\cos x$ in the domain $0\le x\le\frac{\pi}{2}$ and shade the region bounded only by these curves. **2**



(iii) Find the area of the shaded region described in (ii). **3**

$$A=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\sin 2x-\cos x\right)dx$$

$$=\left[-\frac{1}{2}\cos 2x-\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

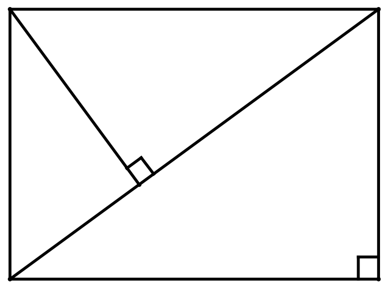
$$=\left(-\frac{1}{2}\cos\pi-\sin\frac{\pi}{2}\right)-\left(-\frac{1}{2}\cos\frac{\pi}{3}-\sin\frac{\pi}{6}\right)$$

$$=\left(\frac{1}{2}-1\right)-\left(-\frac{1}{4}-\frac{1}{2}\right)$$

$$=\frac{1}{4}$$

Area $=\frac{1}{4}\ u^2$

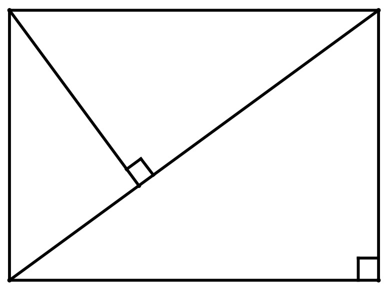
(b)



(Diagram not drawn to scale)

$ABCD$ is a rectangle with $AB = 12\ cm$, $BC = 9\ cm$ and AM perpendicular to BD.

(i) Copy the diagram onto your answer sheet and find the length of BD. **1**



(Diagram not drawn to scale)

$BD^2 = DC^2 + CB^2$ (Pythagoras' Theorem)

$= 12^2 + 9^2$

$= 225$

$BD = 15\ cm$

(ii) Prove that ΔABM is similar to ΔBDC. **3**

$AB \parallel DC$ (opposite sides of a rectangle are parallel)

In ΔABM and ΔBDC

$\angle ABM = \angle BDC$ (alternate angles are equal as $AB \parallel DC$)

$\angle AMB = \angle DCB$ (both $90°$)

$\Delta ABM \parallel\!| \Delta BDC$ (equiangular)

(iii) Hence find the length of BM. (Give reasons) **2**

$\frac{BM}{12} = \frac{12}{15}$ (ratios of corresponding sides are equal in similar triangles)

$BM = 9.6\ cm$

Question 7 (START A NEW PAGE)

Marks

(a) In an experiment, the amount of crystals, x grams, that dissolved in a solution after t minutes was given by:

$$x = 200\left(1 - e^{-kt}\right)$$

(i) After 3 minutes it was found that 50 grams of crystals had dissolved, hence find the value of k. **2**

when $t = 3, x = 50$

$50 = 200\left(1 - e^{-3k}\right)$

$0.25 = 1 - e^{-3k}$

$e^{-3k} = 0.75$

$-3k = \ln(0.75)$

$k = -\dfrac{\ln(0.75)}{3} \quad (\approx 0.09589)$

(ii) At what rate were the crystals dissolving after 5 minutes? Give your answer to the nearest gram/minute. **3**

$x = 200\left(1 - e^{-kt}\right)$

$\dfrac{dx}{dt} = 200ke^{-kt}$ where $k = -\dfrac{\ln(0.75)}{3}$

when $t = 5$

$\dfrac{dx}{dt} = 200\left(-\dfrac{\ln(0.75}{3}\right)e^{5\left(\frac{\ln(0.75}{3}\right)}$

≈ 30.9779566

rate $= 31$ grams/minute

(b) Two particles A and B start from the origin at the same time and move along a straight line so that their velocities in m/s at any time t seconds are given by:

$$v_A = t^2 + 2 \text{ and } v_B = 8 - 2t$$

(i) Show that the two particles never move with the same acceleration. **3**

$$a_A = \frac{dv_A}{dt}$$
$$= 2t$$
$$a_B = \frac{dv_B}{dt}$$
$$= -2$$

if $a_A = a_B$

$2t = -2$

$t = -1$

but $t \geq 0$

$\therefore$ no solution

$\therefore$ particles never move with the same acceleration

(ii) Write a formula for the position of particle A at time t seconds. **2**

$x_A = \frac{1}{3}t^3 + 2t + c$

when $t = 0,\ x_A = 0 \Rightarrow c = 0$

$x_A = \frac{1}{3}t^3 + 2t$

(iii) After leaving the origin, find when the two particles are again at the same position. **2**

$x_B = 8t - t^2 + d$

when $t = 0,\ x_B = 0 \Rightarrow d = 0$

$x_B = 8t - t^2$

when $x_A = x_B$

$\frac{1}{3}t^3 + 2t = 8t - t^2$

$t^3 + 3t^2 - 18t = 0$

$t(t+6)(t-3) = 0$

$t = 3 \quad (t > 0)$

$\therefore$ at same position after 3 seconds

Question 8 (START A NEW PAGE)

(i) Given that $f(x) = x^2\sqrt{10-x}$, show that $f'(x) = \dfrac{5x(8-x)}{2\sqrt{10-x}}$. **3**

$$f(x) = x^2(10-x)^{\frac{1}{2}}$$
$$= (2x)\left\{(10-x)^{\frac{1}{2}}\right\} + (x^2)\left\{\tfrac{1}{2}(10-x)^{-\frac{1}{2}} \times -1\right\}$$
$$= 2x\sqrt{10-x} - \frac{x^2}{2\sqrt{10-x}}$$
$$= \frac{4x(10-x) - x^2}{2\sqrt{10-x}}$$
$$= \frac{40x - 5x^2}{2\sqrt{10-x}}$$
$$= \frac{5x(8-x)}{2\sqrt{10-x}}$$

(ii) State the domain of the function $y = x^2\sqrt{10-x}$. **1**

$Domain: 10 - x \geq 0$

$\therefore \quad x \leq 10$

(iii) Find all the stationary points on the curve $y = x^2\sqrt{10-x}$ and determine their nature. **5**

for stat .pts $\dfrac{dy}{dx} = 0$

$\therefore \quad \dfrac{5x(8-x)}{2\sqrt{10-x}} = 0$

$x = 0 \text{ or } 8$

$x = 0,\ y = 0 \text{ and } x = 8,\ y = 8^2\sqrt{10-8}$

$\qquad\qquad\qquad\qquad = 64\sqrt{2}$

stat. pts. are $(0,0)$ and $(8, 64\sqrt{2})$

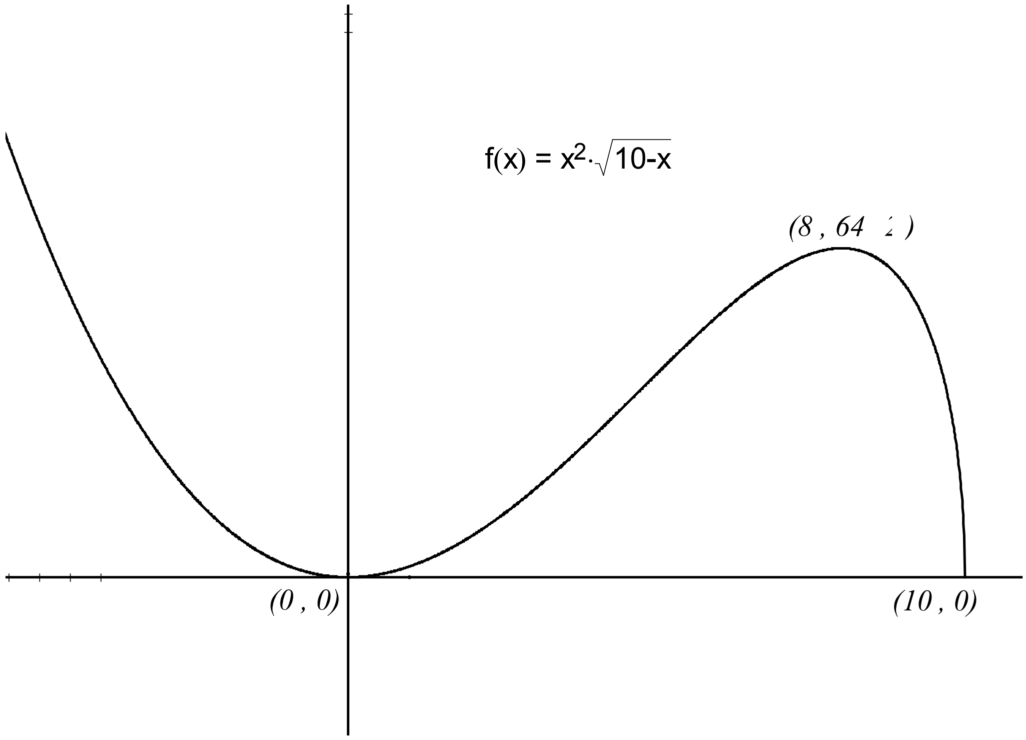
Test nature

x	-1	0	1
$\dfrac{dy}{dx}$	$-\dfrac{45}{2\sqrt{11}}$	0	$\dfrac{35}{6}$
	< 0		> 0
	\ _ / local min.		

x	7	8	9
$\dfrac{dy}{dx}$	$\dfrac{35}{2\sqrt{3}}$	0	$-\dfrac{45}{2}$
	> 0		< 0
	/ ‾ \ local max.		

$\therefore$ local min. tp. at $(0,0)$ and local max. tp. at $(8, 64\sqrt{2})$

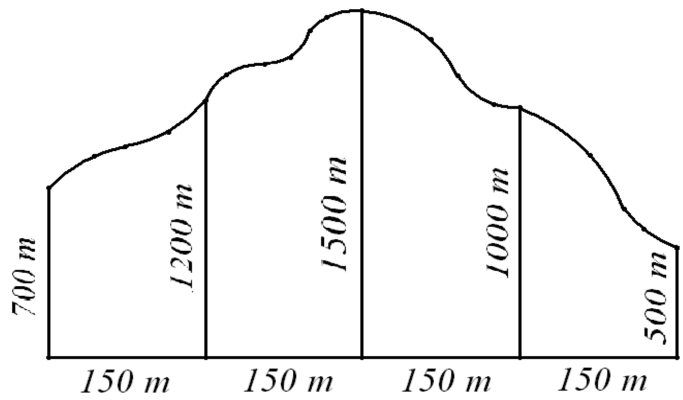
(iv) Sketch the curve $y = x^2\sqrt{10-x}$ indicating its intercepts with the co-ordinate axes and the position of all stationary points. **3**



Question 9 (START A NEW PAGE)

Marks

(a) Prime land along a foreshore is to be reclaimed and developed as part of a housing estate. A plan of the land to be reclaimed is shown below: **3**



(Diagram not drawn to scale)

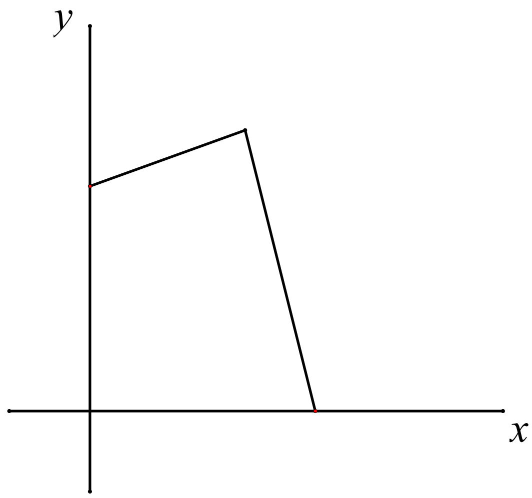
Use Simpson's rule with five function values to find the approximate area, expressed in hectares, of the land to be reclaimed.

$$\text{area} \approx \frac{300}{6}(700 + 4\times1200 + 1500) + \frac{300}{6}(1500 + 4\times1000 + 500) \quad m^2$$

$$\approx 650000 \;\; m^2$$

$$\text{area} \approx 65 \;\; ha$$

(b)



The point $P(x, y)$ moves so that its distance from the point $M(3,0)$ is always twice its distance from the point $N(0,3)$.

(i) Show that the equation of the locus of the point $P(x, y)$ is $x^2 + 2x + y^2 - 8y + 9 = 0$. **2**

$PM = 2PN$

$PM^2 = 4PN^2$

$(x-3)^2 + y^2 = 4\{x^2 + (y-3)^2\}$

$x^2 - 6x + 9 + y^2 = 4(x^2 + y^2 - 6y + 9)$

$x^2 - 6x + 9 + y^2 = 4x^2 + 4y^2 - 24y + 36$

$3x^2 + 6x + 3y^2 - 24y + 27 = 0$

$x^2 + 2x + y^2 - 8y + 9 = 0$

(ii) Give a geometric description of the locus. **2**

$x^2 + 2x + y^2 - 8y + 9 = 0$

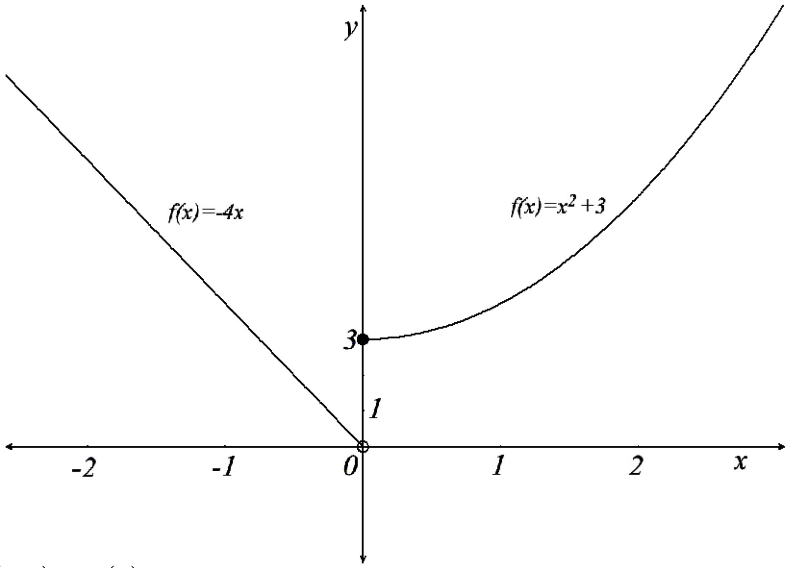
$x^2 + 2x + y^2 - 8y = -9$

$x^2 + 2x + 1 + y^2 - 8y + 16 = -9 + 1 + 16$

$(x+1)^2 + (y-4)^2 = 8$

locus is a circle with centre at $(-1,4)$ and radius $= 2\sqrt{2}$

(c) (i) Graph $y = f(x)$ if $f(x) = \begin{cases} -4x & \text{for } x < 0 \\ x^2 + 3 & \text{for } x \geq 0 \end{cases}$ **2**



(ii) Evaluate $f(-2) + f(2)$. **1**

$$f(-2) + f(2) = \{-4(-2)\} + \{(2)^2 + 3\}$$
$$= 8 + 7$$
$$= 15$$

(iii) Solve for $a > 0 : f(a) = f(-a)$. **2**

$a^2 + 3 = 4a$

$a^2 - 4a + 3 = 0$

$(a-1)(a-3) = 0$

$a = 1 \text{ or } 3$

Question 10 (START A NEW PAGE)

Marks

(a) Abigale decides to save for a car. She creates a savings account into which she deposits \$100 at the beginning of each week. The account pays interest at a rate of 0.16% per week calculated on the value of the account at the end of each week.

(i) Show that Abigale's account is worth $62600\left(1.0016^n - 1\right)$ dollars at the end of n weeks. **3**

Let A_n dollars be the value of Abigale's account at the end of n weeks

$$\mathrm{A}_1 = 100 \times 1.0016$$

$$\begin{aligned}\mathrm{A}_2 &= \left(\mathrm{A}_1 + 100\right) \times 1.0016 \\ &= 100 \times 1.0016^2 + 100 \times 1.0016 \\ &= 100\left(1.0016^2 + 1.0016\right)\end{aligned}$$

$$\begin{aligned}A_3 &= \left(A_2 + 100\right) \times 1.0016 \\ &= 100\left(1.0016^3 + 1.0016^2 + 1.0016\right)\end{aligned}$$

$$\begin{aligned}\mathrm{A}_n &= 100\left(1.0016^{\mathrm{n}} + 1.0016^{\mathrm{n-1}} + \ldots + 1.0016^2 + 1.0016\right) \\ &= 100 \times \frac{1.0016\left(1.0016^n - 1\right)}{1.0016 - 1} \\ &= \frac{100.16\left(1.0016^n - 1\right)}{0.0016}\end{aligned}$$

$$\mathrm{A}_n = 62600\left(1.0016^n - 1\right)$$

(ii) Find the value of Abigale's account at the end of 52 weeks. Give your answer correct to the nearest dollar. **1**

$$\mathrm{A}_n = 62600\left(1.0016^n - 1\right)$$

when $n = 52$

$$\begin{aligned}\mathrm{A}_{52} &= 62600\left(1.0016^{52} - 1\right) \\ &\approx 5426.60\end{aligned}$$

Value of account = \$5427 (to the nearest dollar)

(iii) Find the number of weeks needed for Abigale's account to accumulate \$20000. (Give your answer to nearest week) **3**

$$20000 = 62600(1.0016^n - 1)$$

$$1.0016^n - 1 = \frac{20000}{62600}$$

$$1.0016^n = \frac{413}{313}$$

$$n\ln(1.0016) = \ln\left(\frac{413}{313}\right)$$

$$n = \frac{\ln\left(\frac{413}{313}\right)}{\ln(1.0016)}$$

$$\approx 173.416$$

$\therefore$ no. week $= 173$

(*Note*: 174 weeks is possibly a more correct answer since at 173 weeks the account would be slighty less than \$20000)

(b) When a ship is travelling at a speed of v km/hr, its rate of consumption of fuel in tonnes per hour is given by $125 + 0.004v^3$.

(i) Show that on a voyage of 5000km at a speed of v km/hr the formula for the total fuel used, T tonnes, is given by: **1**

$$T = \frac{625000}{v} + 20v^2$$

$$\text{time} = \frac{5000}{v}$$

Amt. of fuel = time × consumption rate

$$T = \frac{5000}{v} \times (125 + 0.004v^3)$$

$$T = \frac{625000}{v} + 20v^2$$

(ii) Hence find the speed for the greatest fuel economy and the amount of fuel used at this speed. (Justify your answer.) **4**

$$T = \frac{625000}{v} + 20v^2$$

$$\frac{dT}{dv} = -\frac{625000}{v^2} + 40v$$

for stat. pt. $\frac{dT}{dv} = 0$

$$-\frac{625000}{v^2} + 40v = 0$$

$$40v = \frac{625000}{v^2}$$

$$v^3 = 15625$$

$$v = 25$$

$$\frac{d^2T}{dv^2} = \frac{1250000}{v^3} + 40$$

when $v = 25$, $\frac{d^2T}{dv^2} = \frac{1250000}{25^3} + 40$

$$> 0$$

$\therefore$ concave up, $\therefore$ local min. tp.

Since the function is continuous for $v > 0$ and there is only one stat. pt.then the local min. tp. is the absolute minimum.

$\therefore$ minimum speed = 25 km/hr and amount fuel used = 37500 tonnes

This is the END of the examination paper